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REDUCTION OF THE DYNAMICAL APERTURE DUE TO TUNE MODULATION

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1. Introduction

In the 1988 SPS Dynamical Aperture Experiment, strongly excited sextupoles in the SPS were used to study the boundary of stability due to nonlinearities.

It could be shown[1,2] that short-term (i.e. few seconds) particle losses were well predicted by tracking simulations, defining a short-term dynamical aperture. However, an unexpected diffusion rate of the order of millimetres per minute was observed inside this short-term aperture. A complete simulation study of this diffusion would require at least several billions of particle-turns, and therefore is beyond present-day computer capabilities. Furthermore, the precise cause of this diffusion has not yet been pinpointed.

In this note we focus on only one potential candidate for causing this diffusion, namely low-frequency ripple of the magnet power supplies. Our program, used in the earlier tracking simulations, has been suitably augmented to model this ripple. We use as input the experimentally measured ripple depth at 600 Hz of the SPS quadrupole magnets and study only beams coasting through the SPS lattice in the presence of eight strongly excited sextupole.

We typically track 2 particles that start out very close to each other in phase space, for 10^5 turns, with spot checks to 10^6 turns, and determine the amplitude beyond which the motion is chaotic; this defines the dynamical aperture. While a prediction of the diffusion rate is well beyond the scope of this note, at least we are able to study the dependence of the dynamical aperture on the ripple frequency[3].

In general, we observe a substantial decrease in the dynamical aperture relative to the non-ripple case. We varied a number of parameters in order to exclude the possibility that this decrease was due to specific values. At a fixed ripple frequency and depth we studied three different working points and two settings of the distortion sextupoles which suppress or enhance third-order resonances. We also verified that the results are insensitive to the number of ripple elements in our simulation, provided, naturally, that their integrated effect on the tune modulations is kept constant.

2. Influence of Ripple on the Dynamical Aperture

A major result of the 1988 Dynamical Aperture Experiment is that a short-term dynamical aperture can be defined, which corresponds to a particle loss after a few seconds, and that it can easily be predicted by finding the onset of chaotic motion by tracking. This can be found using SIXTRACK[4] by studying the behaviour of the distance in phase space of two initially close particles as a function of time; figure 1[2] is one example of how well the short-term dynamical aperture can be predicted. The experiment, however, shows that well inside the short-term stability border one finds considerable diffusion rates of 6 and 3 mm per minute (see Ref. 1 for details). This diffusion cannot be explained by the tracking, in which the linear lattice is only perturbed by sextupoles; all studied amplitudes below the short-term dynamical aperture are of regular motion type, which means that the motion is indefinitely stable; secondly, long-term tracking (10^6 turns corresponding to 23 sec) with PATRAC[5] shows that just inside the short-term dynamical aperture there is no amplitude increase whatsoever, while for the experiment one expects at least an amplitude increase of 2.3 mm (6mm/min. x 23/60 min.). From this we conclude, since the strong effects are well predicted by tracking, that one has to include some additional small effect in the tracking model that may lead to this diffusion. One candidate for such a diffusion process is the ripple of the power supplies of the magnets, which is dangerous as it moves the working point in the Q_z-Q_x diagram. A 600 Hz ripple of the main quadrupoles was measured[6] with the following strengths:

$$\Delta K_f(\text{focusing}) = \pm 1.1 \times 10^{-6} [m^{-2}] \tag{1}$$

$$\Delta K_d(\text{defocussing} = \pm 0.44 \times 10^{-6} [m^{-2}]$$

which corresponds to a relative change in strength:

$$\frac{\Delta K_F}{K_F} = \pm 7.5 \times 10^{-5}$$

$$\frac{\Delta K_D}{K_D} = \pm 3.0 \times 10^{-5} \tag{2}$$

This variation in the strength of the quadrupoles leads to a tune modulation of depth:

$$\Delta Q_x = \pm 3.6 \times 10^{-3}$$

$$\Delta Q_x = \pm 1.9 \times 10^{-3} \tag{3}$$

The frequency of 600 Hz which is a strong peak in the ripple frequency spectrum, is due to the 12-phase converter of the power supplies. But there are also other peaks at lower frequency, for instance at all multiples of 50 Hz and there is even a 10 Hz ripple which is caused by some resonance of the power supply regulation loops.

We have studied the dependence of the stability of the beam over a wide range of ripple frequencies* with the modulation depth fixed to

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*To determine the nature of the motion (regular or chaotic) 20 000 turns are sufficient when no ripple is considered, but once ripple is introduced a less violent chaotic behaviour is found at smaller amplitudes, so that 10^6 turns are needed for an unambiguous distinction between regular and chaotic motion. As one turn for two particles for the SPS ring takes about 3 ms on the IBM (CERN) and 18 ms on the VAX (LBL), 10^6 turns corresponds to 50 minutes and 5 hours respectively. We therefore decided to restrict ourselves to 10^5 turns at the LBL-VAX. All results of the next chapter have therefore an uncertainty of about ± 1 mm. For the data in fig. 2, however, the necessary 10^6 turns were used.

the value corresponding to 600 Hz stated above (fig. 2). To understand qualitatively why at small frequencies the dynamical aperture reduces dramatically, let's look at figs. 3 and 4, which show the normalized FFT spectrum of the x-coordinate (8192 turns) for 600 and 50 Hz respectively. At 600 Hz one finds, besides the main peak, two rather small sideband peaks at a distance of 0.0138 (600 Hz × 23 μs).

The relative sizes of the peaks are given by[7]:

$$J_n(\varepsilon) \approx \frac{\varepsilon}{2} \quad ; \quad J_n = \text{Bessel function}$$
$$\varepsilon = \frac{\Delta\omega}{\omega_s} \ll 1 \quad ; \quad \omega_s = \text{ripple frequency} \; ; \; \Delta\omega = \text{ripple depth} \tag{4}$$

At ω_s = 600 Hz the sideband peaks should have a relative height of

$$\frac{\varepsilon}{2} = \frac{\Delta\omega}{2\omega_s} = 0.13 \tag{5}$$

which is indeed found in the tracking (fig. 3). Below 160 Hz the condition of Eq.4 is no longer fulfilled. In such a case (fig. 4 at 50 Hz < 160 Hz), one finds numerous sidebands in an interval of ± $\Delta\omega$ around the main peak, that no longer has to be the dominant peak.

Our conclusion is that the concept of a distinct value of the tune is no longer useful in this situation, considering time intervals large compared to a ripple period, (e.g. 50 Hz := 868 turns). Instead, one has to consider the tune as being smeared out over the whole interval ± $\Delta\omega$. An alternative way of expressing this is to consider that resonances in this range overlap, causing chaotic motion even at low amplitude.

3. Studying the Reduction of the Dynamical Aperture for Various Sets of Parameters

Systematic studies were done at the chosen ripple depth and with the ripple frequency fixed to 50 Hz to exclude that the drop in the dynamical aperture is due to a specific choice of the parameters. Three working points were studied (fig. 5): Working point number one is in the vicinity of the strong 5th order resonances (the amplitude dependence moves the tune to the left when the amplitudes are increased), where we find only a tiny region of chaotic motion at the dynamical aperture when no ripple is involved (this working point was studied in the last chapter). Working point 2 is close to the 7th order resonances, which lead even without ripple to a wider range of chaotic motion due to a nest of weak resonances (see Ref. 2 for details). Finally working point 3 is the nominal working point for the SPS.

Secondly, the influence of the polarity of the power supplies of the sextupoles are investigated. In the SPS experiment 2 different sets of polarities are used: the first denoted with (++++−−−−) suppresses the resonances excited by sextupoles in first order, which are the 3rd and 1st order resonances (this polarity was used in the last chapter). These resonances are strongly excited with the second set of polarities (+−−++−−+).

Figure 6 shows the smear versus the dynamical aperture without a ripple being considered, in each of the six cases the largest amplitude with regular motion and smallest amplitude with chaotic motion is displayed ranging from 8 to 22 mm. In cases A and E one finds a large change in the smear. This is typical for strong resonances (5th order in our case), which make the motion abruptly and strongly chaotic, which in turn leads to large fluctuations in the amplitude. Figure 7 shows the corresponding situation with a 50 Hz ripple: in all cases the dynamical aperture goes down by about a factor of 2 (it ranges between 4 and 11 mm) and now even in cases A and E the motion is no longer dominated by a single strong resonance. A large drop in the dynamical aperture due to ripple is therefore to be expected in general: it is not due to a bad choice of parameters.

Another question is whether approximating the collective ripple of all magnet elements by one rippling element in each plane is a valid thing to do.

We find that a change from 2 to 20 rippling elements leaves the dynamical aperture approximately unchanged. However, the change of the number of rippling elements has some effect, as is shown in the FFT signal of the x-coordinate for both cases (figs. 8 and 9): the peaks lie at the same location but the relative heights have changed.

Finally we would like to present an interesting relation between the smear and the tuneshift. In Ref. 8 it has been shown that for the polarity of sextupoles, where the first order sextupole resonances are suppressed (++++----), one finds that the tuneshift grows linearly with smear (Fig. 10). For the same parameters one also finds this linear dependence, when the polarity is changed to a situation, where the first order sextupole resonances are excited (+--++--+) (Fig. 11).

4. Conclusion

Including ripple of the power supplies of the magnets in the tracking model has a strong effect on the long-term dynamical aperture, while it only slightly changes the short-term dynamical aperture. This reduction in the dynamical aperture strongly increases when the ripple frequency is reduced, which has been explained qualitatively.

A change in the choice of the working point and polarity of sextupoles left the relative drop in the dynamical aperture approximately unchanged. Representing the collective ripple of all magnets by 2 localised quadrupoles has shown to be a good quantitative approximation. We therefore conclude that the ripple is a good candidate to explain the diffusion found in the experiment. For a more quantitative comparison between experiment and tracking it is, however, necessary to do an experiment where the ripple is dominated by a magnet element of controlable ripple frequency and amplitude. We strongly suggest such an experiment, to clarify the question as to whether the diffusion is indeed due to power supply ripple.

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2. F. Schmidt, Tracking results with SIXTRACK for the 1988 Dynamical Aperture Experiment, CERN SPS/88-49 (AMS).

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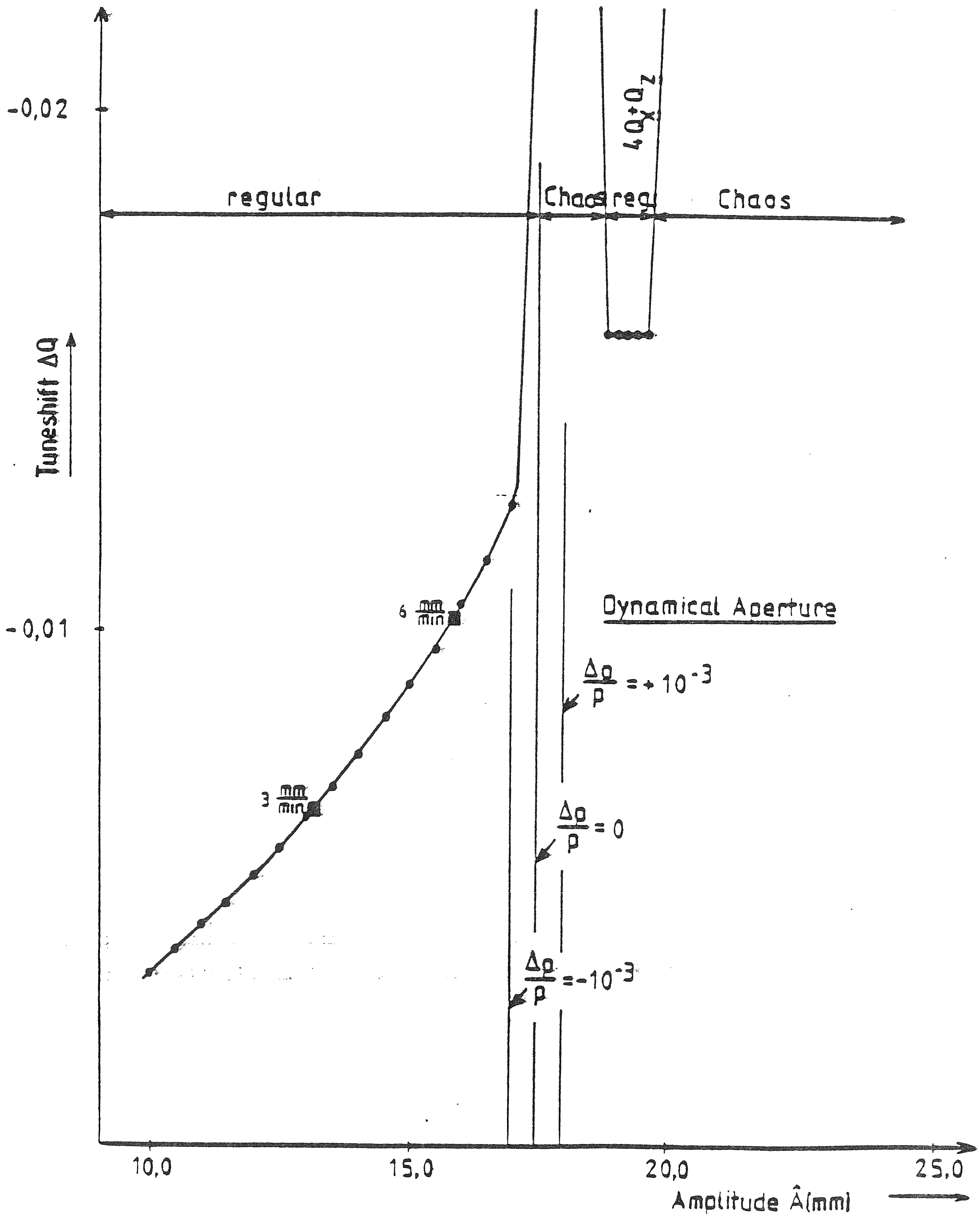
6. J.D. Pahud, private communication.

7. J. Gareyte, private communication.

8. F. Schmidt, Smear Calculation in the Presence of Linear Coupling for the 1988 Dynamical Aperture Experiment, CERN SPS/88-50 (AMS).

Q_{xo} = 26.625
Q_{zo} = 26.560
β_x = 106.7 m
■ measured diffusion rate
• = results from tracking to 10000 turns (see reference 2 for more details).

Fig 1



ripple frequency.

The solid line labelled Dynamical Aperture is the border between the regions where one finds regular and chaotic motion respectively. The error bars show the precision to which this border of stability was determined in the tracking.

+ = particle loss after ≈ 10^3 turns

* = particle loss after ≈ 10^4 turns (numbers in brackets give the precise turn number)

Fig 2

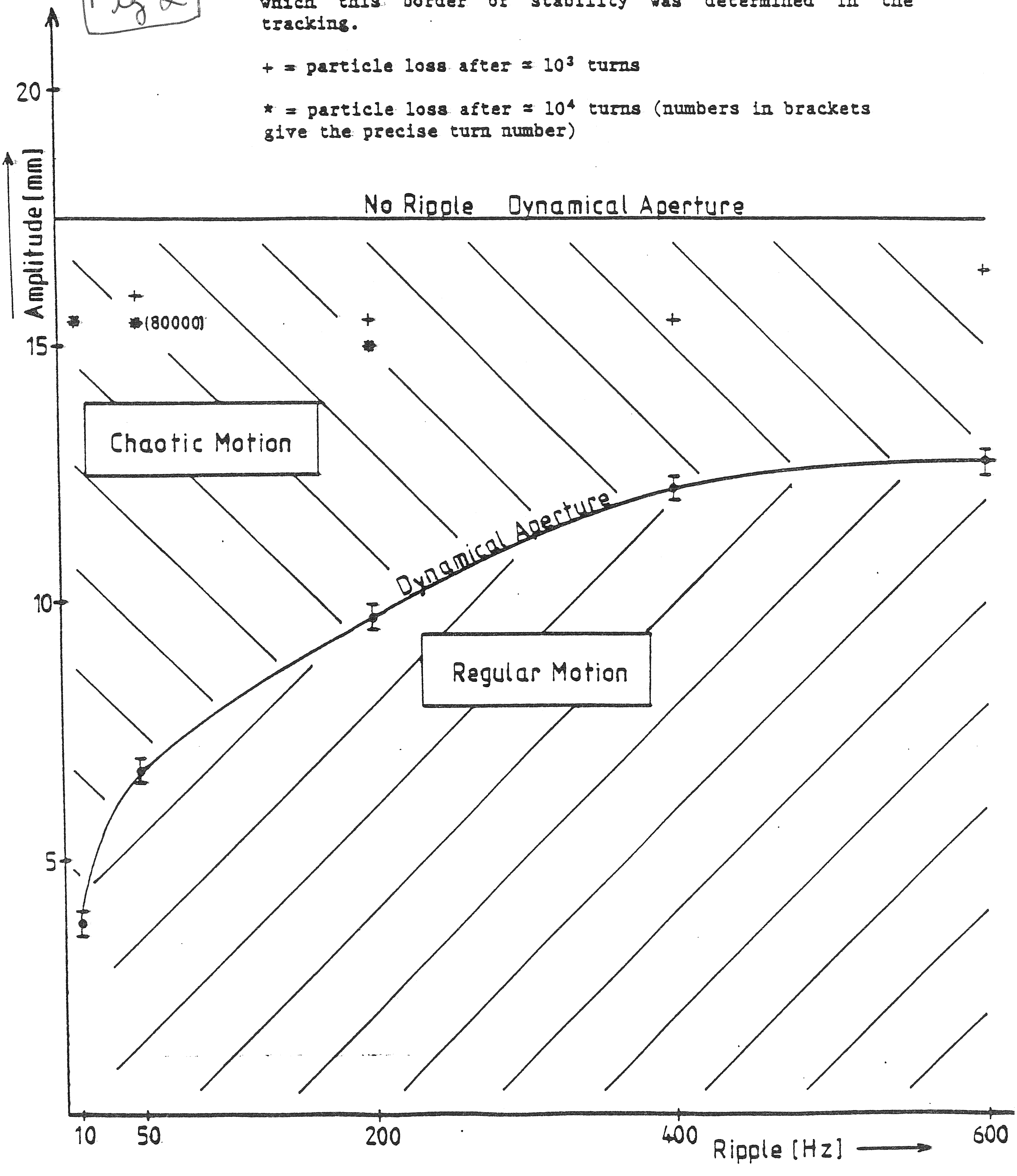


Fig. 3. FFT-spectrum of the x-coordinate with a ripple frequency of 600 Hz

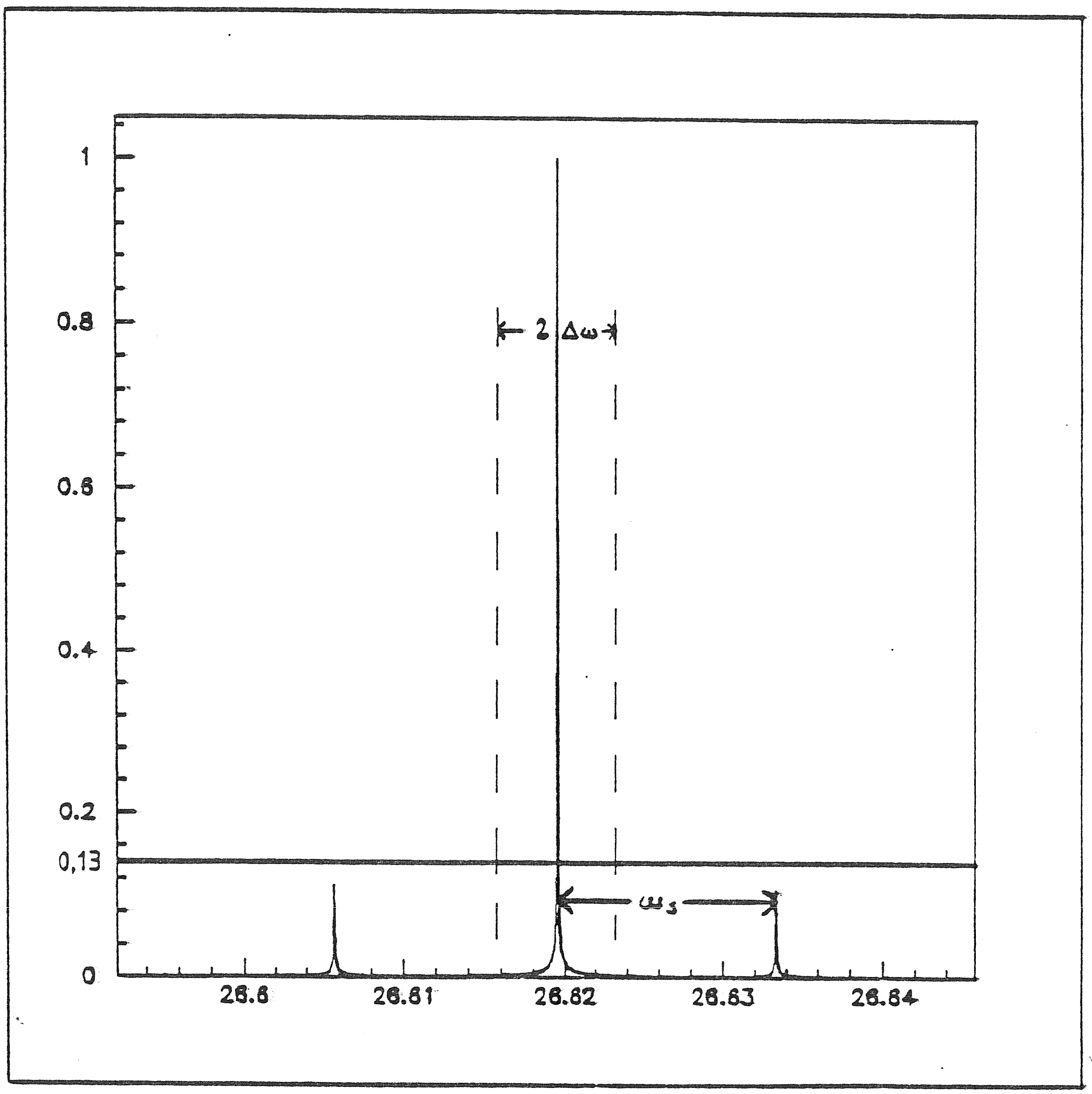


Fig. 4. FFT-spectrum of the x-coordinate with a ripple frequency of 50 Hz.

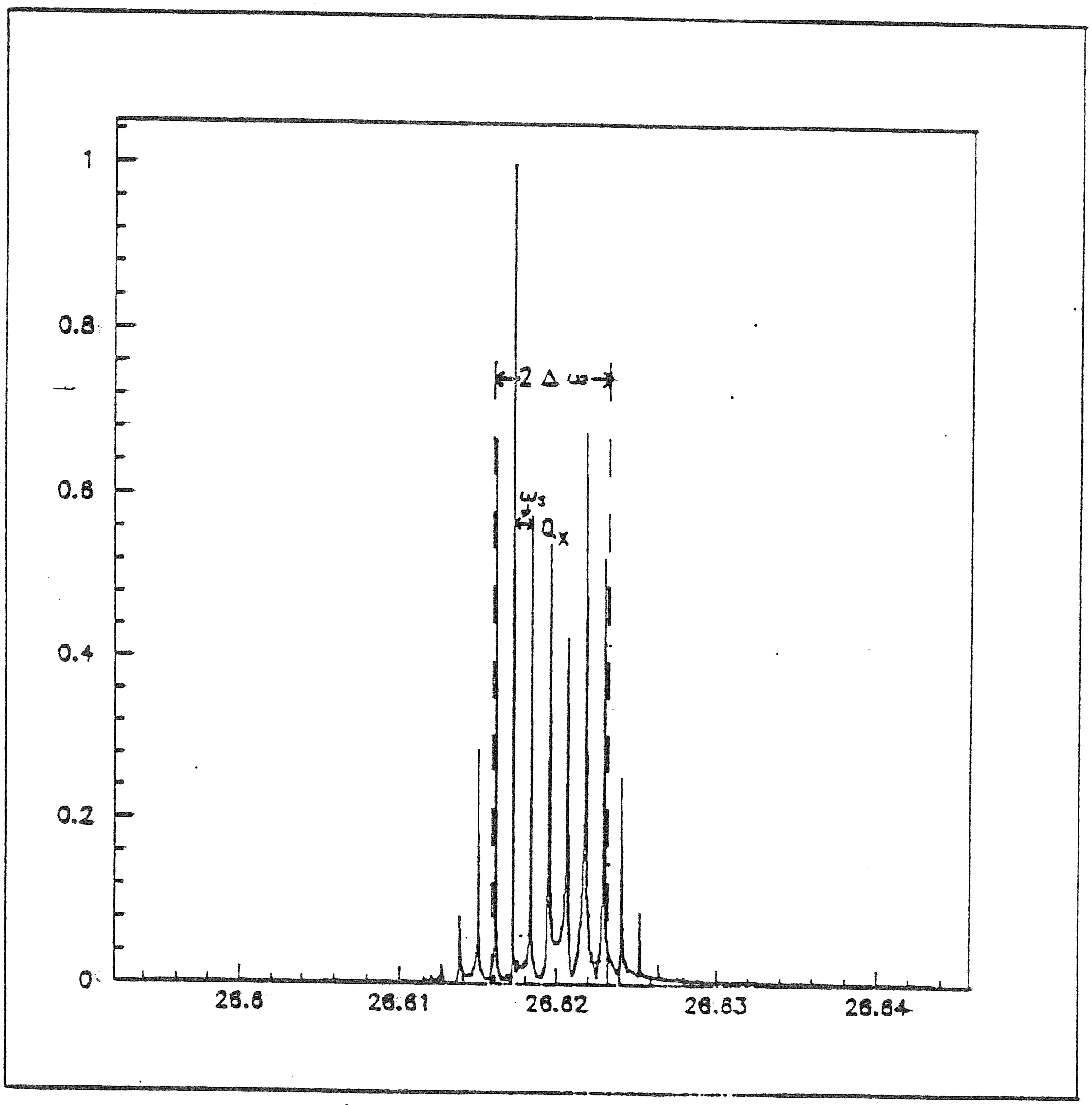


Fig. 5. Working points in the Q_z-Q_x diagram.

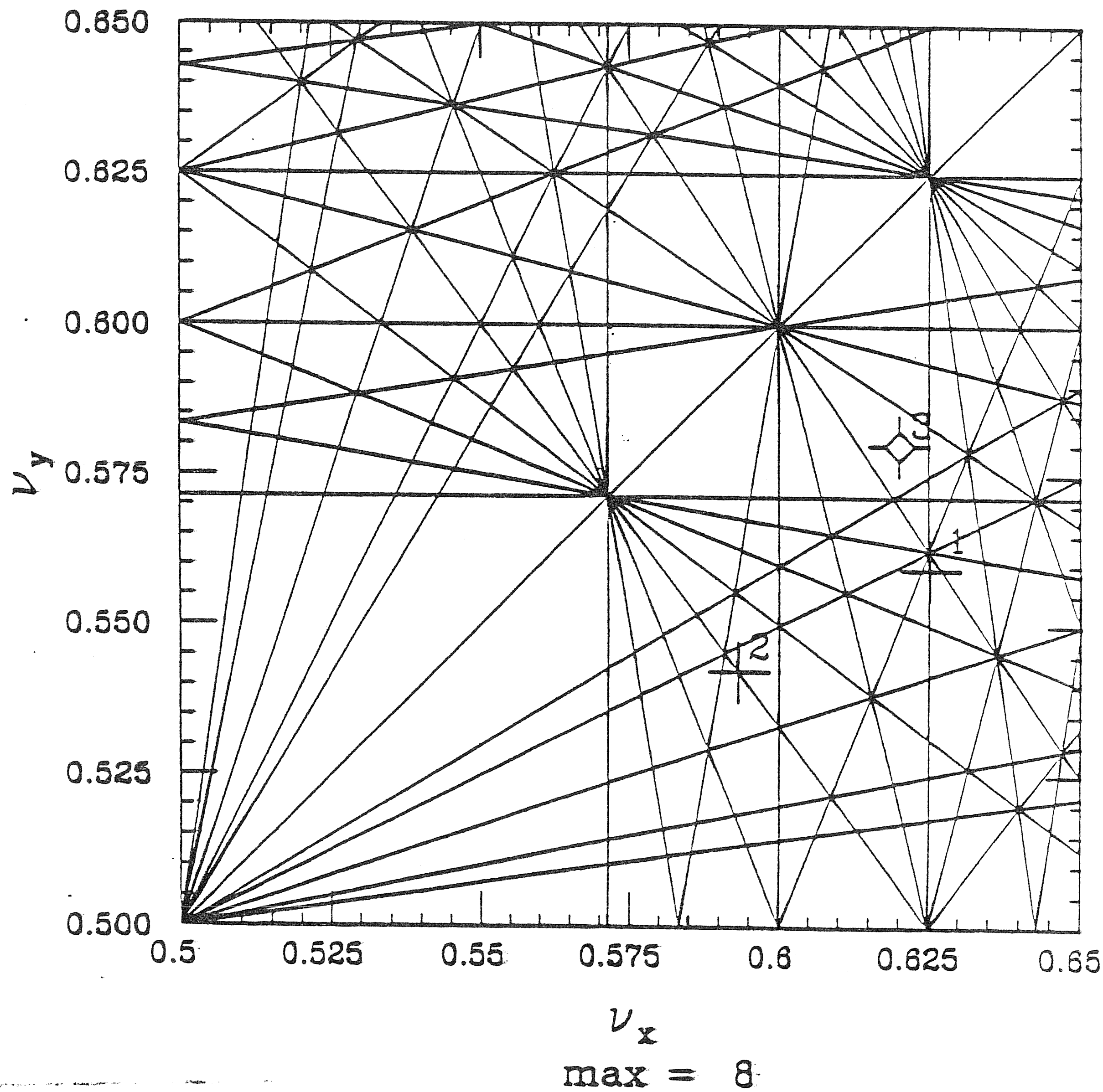


Fig. 6. Dynamical aperture without ripple.

The two crosses stand for the largest amplitude with regular motion and the smallest amplitude with chaotic motion respectively. In cases A and E the presence of a strong resonance makes the smear for the chaotic particle rise strongly.

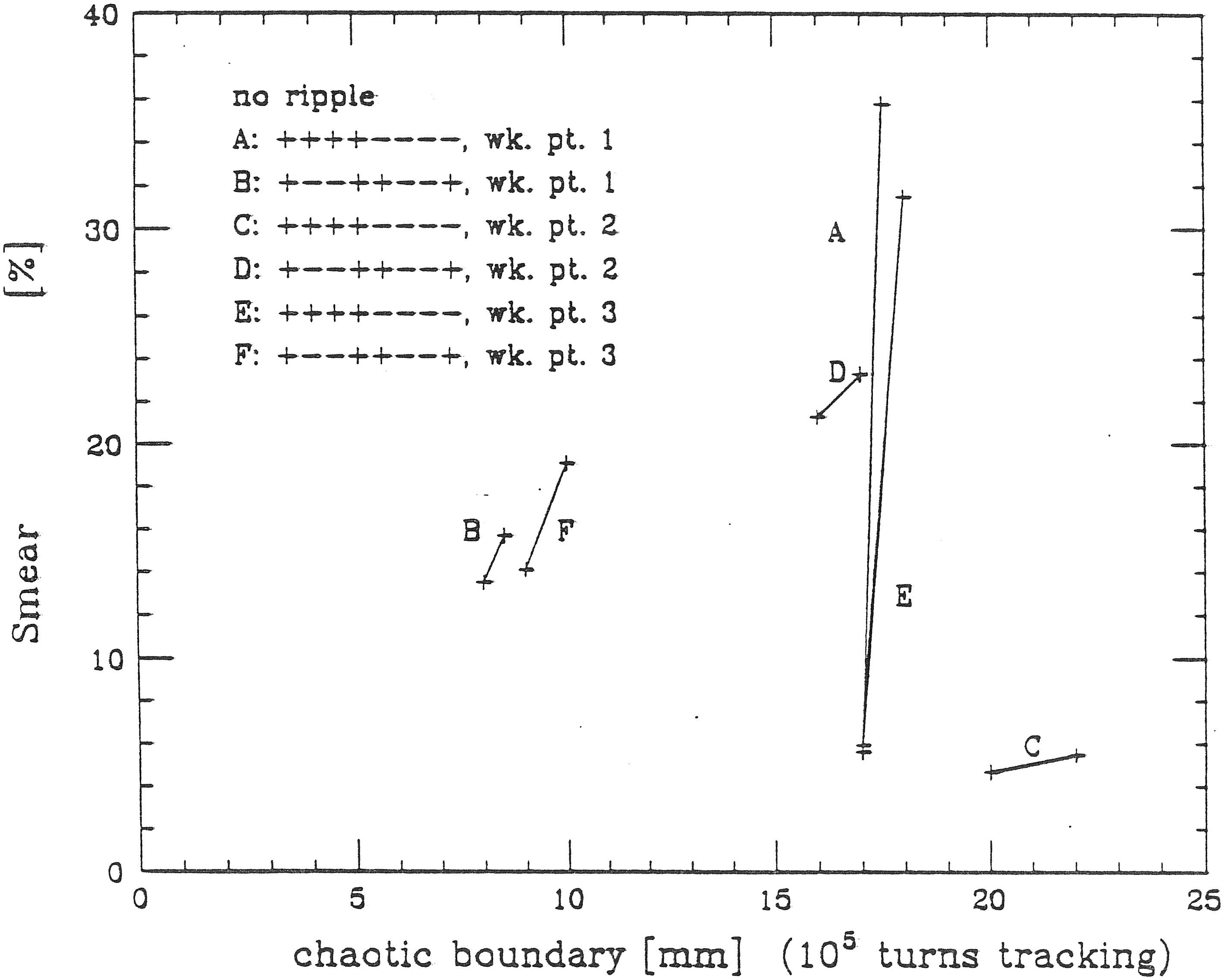


Fig. 7. Dynamical aperture including the ripple.

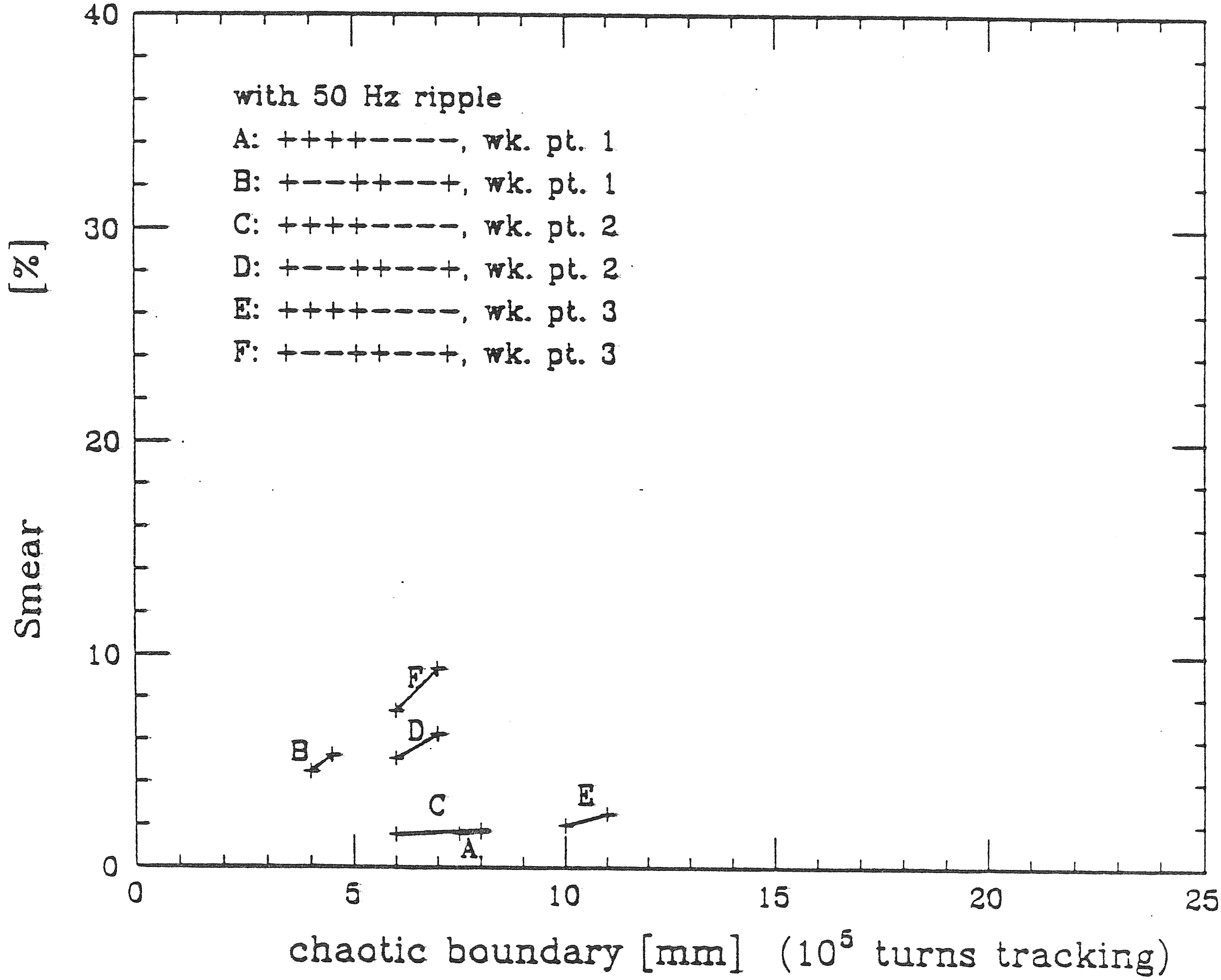


Fig. 8. FFT-spectrum of the x-coordinate for 2 rippling elements.

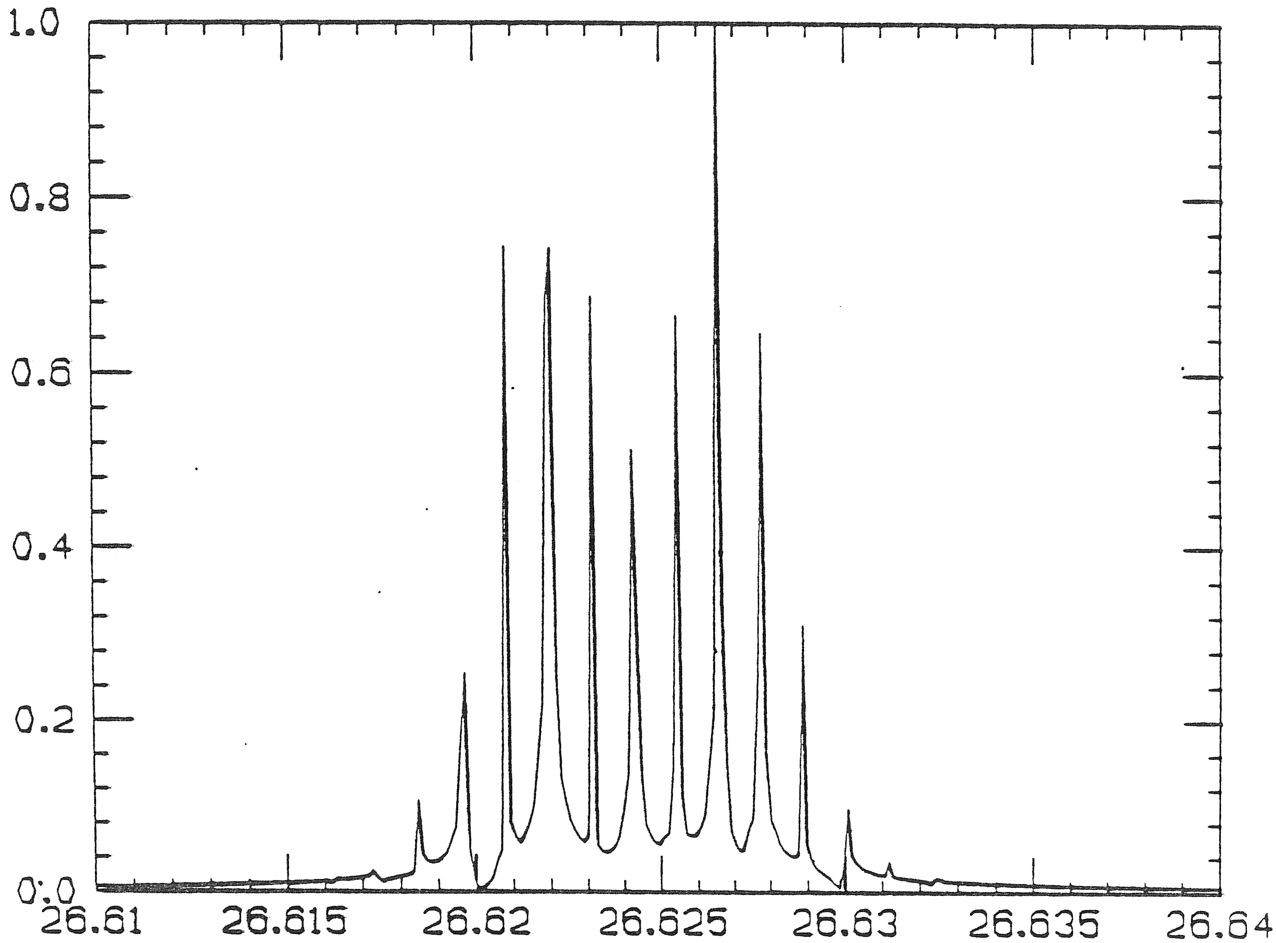


Fig. 9. FFT-spectrum of the x-coordinate for 20 rippling elements.

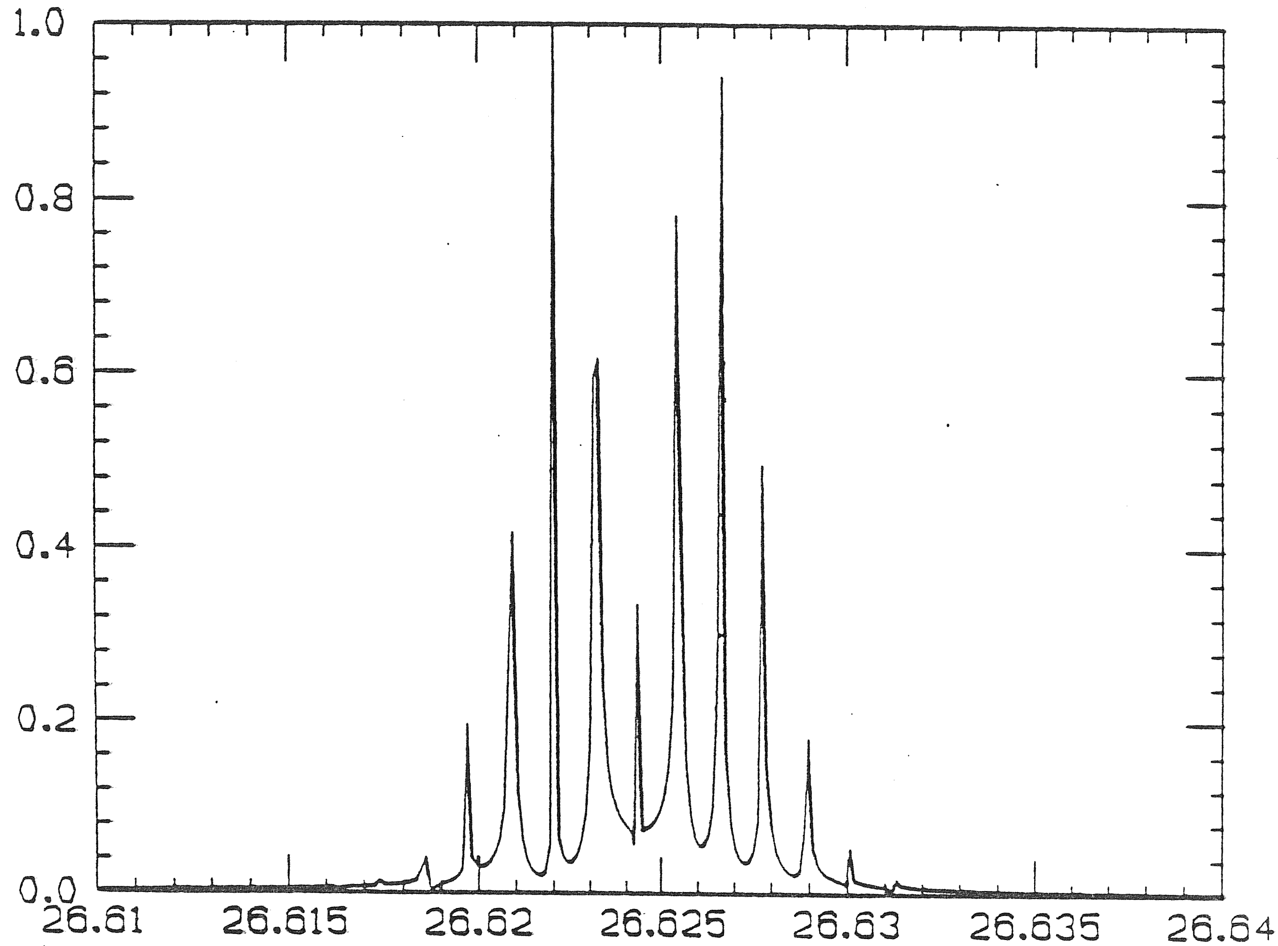


Fig. 10. Tune shift vs. smear from tracking simulations (taken from Ref. 8)

The polarity (++++----) suppresses the first-order sextupole resonances.

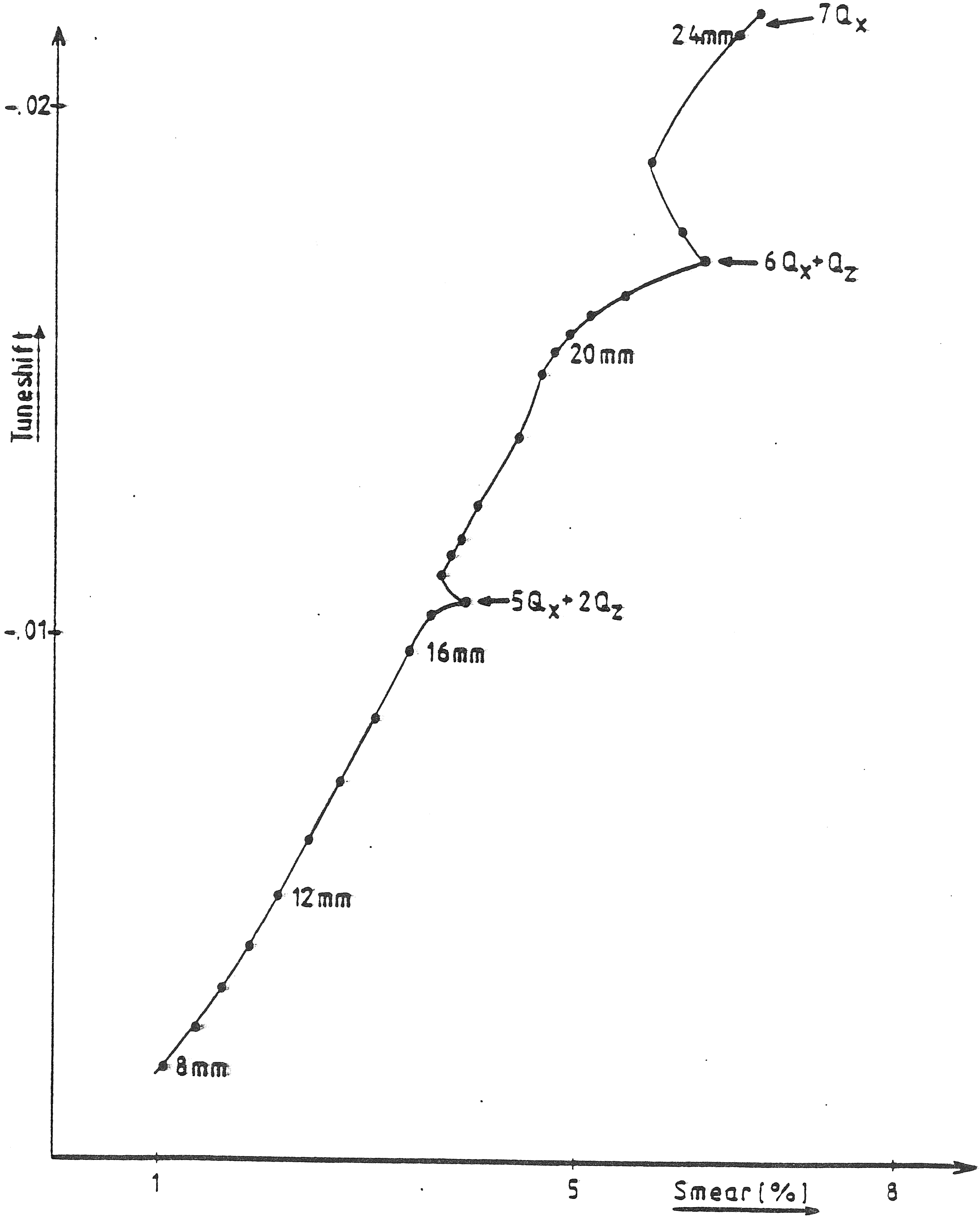


Fig. 11. Tune shift vs. Smear.

The polarity (+−−++−−+) enhances the first-order sextupole resonances.

D.A. = dynamical aperture.